

Open Bisimulation, Revisited

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Outline

- 1 The pi-calculus
- 2 Bisimulations
- 3 The spi-calculus
- 4 K-open bisimulation

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Syntax

- Processes

$$P, Q ::= \mathbf{0} \quad | \quad \pi.P \quad | \quad !P \quad | \quad (\nu z)P \quad | \quad P \parallel Q \quad | \quad [x=y]P \quad | \quad P + Q$$

- Prefixes

$$\pi ::= \tau \quad | \quad x(z) \quad | \quad \bar{x}\langle z \rangle$$

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- Only names

Labelled Semantics

$$\begin{array}{c}
 \text{INPUT} \frac{}{a(x).P \xrightarrow{a(x)} P} \\
 \text{OPEN} \frac{P \xrightarrow{\bar{a}z} P'}{(\nu z)P \xrightarrow{(\nu z)\bar{a}z} P'} \quad z \neq a \\
 \text{CLOSE-L} \frac{P \xrightarrow{a(x)} P' \quad Q \xrightarrow{(\nu z)\bar{a}z} Q'}{P \parallel Q \xrightarrow{\tau} (\nu z)(P'\{z/x\} \parallel Q')} \quad z \notin \text{fn}(P)
 \end{array}$$

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- Proof techniques for showing process equivalence

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$$Q$$

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Some questions when designing a bisimulation:

- When should substitutions be applied?
- Which names are substitutable?
- By what?

Late and open

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Indexed by a *distinction* D .

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In the following, we concentrate on open.

The lazy flavour of open

$$P \stackrel{\text{def}}{=} c(x).(\tau + \tau.\tau + \tau.[x=a]\tau)$$
$$Q \stackrel{\text{def}}{=} c(x).(\tau + \tau.\tau)$$

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- P and Q are late bisimilar but not open
- In open, the instantiation of x can be delayed until x is used
- Open is “very late”

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For these reasons, we wanted to extend open to the spi-calculus.

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- Expressions

$$E, F ::= x \quad | \quad (E.F) \quad | \quad \pi_1(E) \quad | \quad \pi_2(E) \\ \quad \quad \quad | \quad E_F(E) \quad | \quad D_F(E)$$

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- Guards

$$\phi ::= [E = F] \quad | \quad [E : \mathcal{N}]$$

Open in spi?

- Consider

$$P \stackrel{\text{def}}{=} (\nu k) (\nu m) \bar{a}\langle E_k(m) \rangle . a(x) . (\bar{a}\langle k \rangle \parallel [x = k] \bar{a}\langle a \rangle)$$

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 and any message built with these “bricks”

Bisimulations in spi

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For any m and n , we want $P(m)$ and $P(n)$ equivalent.

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- Abadi and Gordon have introduced environment-sensitive bisimulation.

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Different kinds of free names

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 - ▶ any fresh name.

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Environments

$$P \stackrel{\text{def}}{=} (\nu k) \bar{a}\langle k \rangle . a(x) . ((\nu l) \bar{b}\langle l \rangle \| [x = k] \bar{a}\langle a \rangle)$$

C	V	γ
\emptyset	$\{a, b\}$	\emptyset

$$D = \emptyset$$

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- A substitution σ respects e if $\text{supp}(\sigma) \subseteq V$ and σ does not “contradict” \prec
- The corresponding distinction is

$$D(C, V, \prec) \stackrel{\text{def}}{=} C^\neq \cup \{n \neq x \mid n \in C \wedge \neg(n \prec x)\}$$

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$$P \approx_K^{(C, V, \prec)} Q \Rightarrow P \approx_O^{D(C, V, \prec)} Q$$

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- In particular

$$P \approx_O^\emptyset Q \Leftrightarrow P \approx_K^{(\emptyset, \text{fn}(P+Q), \emptyset)} Q$$

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- We conjecture that if D can be expressed in terms of (C, V, \prec) , then \approx_O^D is more than a D -congruence
- We have a proposal for an extension of K-open to spi which is sound w.r.t. barbed equivalence via late hedged bisimulation

Future work

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- Study the congruence properties of K-open

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- Study the congruence properties of K-open
- Study the extension of K-open to spi
 - ▶ Link with symbolic bisimulation of [BBN04]
 - ▶ Congruence properties

Thank you!

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Questions?

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