Open Bisimulation, Revisited

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Outline

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Syntax

- Processes

$$P, Q \quad ::= \quad \mathbf{0} \;\Big|\; \pi.P \;\Big|\; !P \;\Big|\; (\nu z)\,P \;\Big|\; P \| Q \;\Big|\; [x = y]P \;\Big|\; P + Q$$

- Prefixes

$$\pi \quad ::= \quad \tau \;\Big|\; x(z) \;\Big|\; \overline{x}\langle z \rangle$$

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- Only names

Labelled Semantics

$$\textsc{Input}\ \frac{}{a(x).P \xrightarrow{a(x)} P} \qquad \textsc{Open}\ \frac{P \xrightarrow{\bar{a}z} P'}{(\nu z)P \xrightarrow{(\nu z)\bar{a}z} P'}\ z \neq a$$

$$\textsc{Close-L}\ \frac{P \xrightarrow{a(x)} P' \qquad Q \xrightarrow{(\nu z)\bar{a}z} Q'}{P \| Q \xrightarrow{\tau} (\nu z)(P'\{^{z}/_{x}\} \| Q')}\ z \notin \mathrm{fn}(P)$$

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Some questions when designing a bisimulation:

- When should substitutions be applied?
- Which names are substitutable?
- By what?

Late and open

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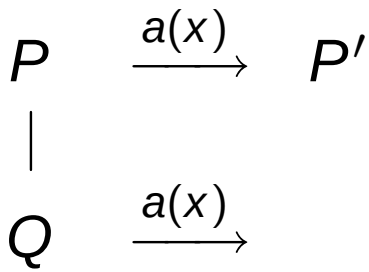

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Indexed by a *distinction* D.

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In the following, we concentrate on open.

The lazy flavour of open

$$
\begin{aligned}
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Q &\stackrel{\text{def}}{=} c(x).(\tau + \tau.\tau)
\end{aligned}
$$

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- Open is “very late”

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For these reasons, we wanted to extend open to the spi-calculus.

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- Messages

$$M, N \quad ::= \quad x \quad \mid \quad (M\,.\,N) \quad \mid \quad \mathrm{E}_N(M)$$

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- Guards

$$\phi \quad ::= \quad [E = F] \quad | \quad [E : \mathcal{N}]$$

Open in spi?

- Consider

$$P \stackrel{\text{def}}{=} (\nu k)\,(\nu m)\,\overline{a}\langle \mathsf{E}_k(m)\rangle.a(x).(\overline{a}\langle k\rangle \| [x=k]\overline{a}\langle a\rangle)$$

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  a, z for any z fresh(not in $\{k, m, a\}$), $\mathsf{E}_k(m)$
  and any message built with these “bricks”

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- Abadi and Gordon have introduced environment-sensitive bisimulation.

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  - any fresh name.

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Environments

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C	V	$\prec$
$\emptyset$	$\{a, b\}$	$\emptyset$

$$D = \emptyset$$

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- The corresponding distinction is

$$D(C, V, \prec) \stackrel{\text{def}}{=} C^{\neq} \cup \{n \neq x \mid n \in C \wedge \neg(n \prec x)\}$$

Some results

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- $$P \approx_{\mathrm{K}}^{(C,V,\prec)} Q \Rightarrow P \approx_{\mathrm{O}}^{D(C,V,\prec)} Q$$

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- $$P \approx_{\mathrm{O}}^{D(C,V,\prec)} Q \Rightarrow P \approx_{\mathrm{K}}^{(C,V,\prec)} Q$$

Some results

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- $$P \approx_{\mathsf{K}}^{(C,V,\prec)} Q \Rightarrow P \approx_{\mathsf{O}}^{D(C,V,\prec)} Q$$
- $$P \approx_{\mathsf{O}}^{D(C,V,\prec)} Q \Rightarrow P \approx_{\mathsf{K}}^{(C,V,\prec)} Q$$
- In particular
$$P \approx_{\mathsf{O}}^{\emptyset} Q \Leftrightarrow P \approx_{\mathsf{K}}^{(\emptyset,\mathsf{fn}(P+Q),\emptyset)} Q$$

Conclusion

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Conclusion

- We have defined K-open bisimulation and proved it coincides with open whenever it is defined
- We conjecture that if D can be expressed in terms of $(C, V, \prec)$, then $\approx_{\mathrm{O}}^{D}$ is more than a D-congruence
- We have a proposal for an extension of K-open to spi which is sound w.r.t. barbed equivalence via late hedged bisimulation

Future work

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- Study the congruence properties of K-open

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- Study the extension of K-open to spi
  - Link with symbolic bisimulation of [BBN04]
  - Congruence properties

Thank you!

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Questions?

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