



A formal study of two physical countermeasures against side channel attacks

PROOFS'2012

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2012, September 13th

- 1. Introduction**
- 2. Combinational Circuits**
- 3. Formalisation of WDDL and BCDL**
- 4. Discussion**
- 5. Conclusion**

- 1. Introduction**
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 1. Language
 2. Formal semantics, equivalences
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 1. Preliminaries
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Physical Attacks

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Extra logic is required in order to mask the sensitive data or to balance the leakage.

Dual-rail Precharge Logic

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$$P, Q ::= 0$$

empty circuit

Syntax

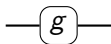
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logical gate $g \in \mathcal{G}$

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a single wire



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a fork

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a swap

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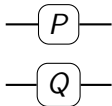
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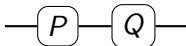
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sequential composition

Well-formedness

Circuit with n inputs and m outputs

$$\frac{\mathcal{T}(g) = (n, m)}{g : n \otimes m} \quad g \in \mathcal{G}$$

$$\overline{\mathbf{0} : 0 \otimes 0}$$

$$\overline{\mathbf{1} : 1 \otimes 1}$$

$$\overline{\mathbf{Y} : 1 \otimes 2}$$

$$\overline{\mathbf{X} : 2 \otimes 2}$$

$$\frac{P_1 : n_1 \otimes m_1 \quad P_2 : n_2 \otimes m_2}{P_1 | P_2 : n_1 + n_2 \otimes m_1 + m_2}$$

$$\frac{P_1 : n \otimes m \quad P_2 : m \otimes p}{P_1 ; P_2 : n \otimes p}$$

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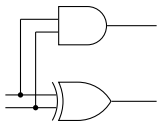
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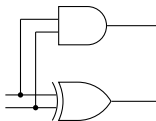
Example: a half-adder

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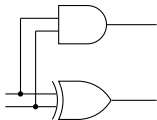
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Half is a combinational circuit with 2 inputs and 2 outputs, i.e.
Half : $2 \otimes 2$.

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Preliminary definitions

Alphabet, words, concatenation.

An **alphabet** Σ is a finite set of letters.

A **word** u over Σ is a finite sequence of letters $u = u_1 \cdots u_n$ where $u_i \in \Sigma$.

The set of words over Σ is noted Σ^* .

The integer n is the **length** of u and noted $|u|$.

The **empty word** is noted ϵ and is the unique word of length 0.

The set of words of length n is noted Σ^n .

The **concatenation** of $u = u_1 \cdots u_n$ and $v = v_1 \cdots v_m$ is defined $u \bullet v := u_1 \cdots u_n v_1 \cdots v_m$.

Formal semantics

C computes y on input x

The semantics of circuits is given by a relation of $\Sigma^* \times \Sigma^*$.

$$\frac{x \in \Sigma^* \quad \mathcal{E}(g)(x) = y \in \Sigma^*}{g \Vdash x \rightsquigarrow y} \quad g \in \mathcal{G}$$

$$\overline{\mathbf{0} \Vdash \epsilon \rightsquigarrow \epsilon}$$

$$\overline{\mathbf{I} \Vdash a \rightsquigarrow a} \quad a \in \Sigma$$

$$\overline{\mathbf{Y} \Vdash a \rightsquigarrow aa} \quad a \in \Sigma$$

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$\mathcal{E}(g)$ is a partial function $\Sigma^* \rightarrow \Sigma^*$, defined consistently w.r.t. the typing function.

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$$P \simeq Q \iff \forall x, y : P \Vdash x \rightsquigarrow y \iff Q \Vdash x \rightsquigarrow y$$

Structural congruence

\equiv identifies circuits that only differ in some minor wiring details.

It is the smallest congruence that satisfies the following equations:

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- If $P_1 : n \otimes m$ and $P_2 : m \otimes p$ then
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- $\mathbf{X} ; (\mathbf{Y} | \mathbf{Y}) \equiv (\mathbf{Y} | \mathbf{Y}) ; (\mathbf{I} | \mathbf{X} | \mathbf{I}) ; (\mathbf{X} | \mathbf{X}) ; (\mathbf{I} | \mathbf{X} | \mathbf{I})$

Some results

- If $P : n \otimes m$ and $P : n' \otimes m'$ then $n = n'$ and $m = m'$.
- If $P \equiv Q$ then $P : n \otimes m \iff Q : n \otimes m$.
- If $P \Vdash x \rightsquigarrow y$ then $P : |x| \otimes |y|$.
- If $P : n \otimes m$ and $P \Vdash x \rightsquigarrow y$ then $|x| = n$ and $|y| = m$.
- If $P : n \otimes m$ then for any x such that $|x| = n$ there exists y such that $P \Vdash x \rightsquigarrow y$.
- If $P \Vdash x \rightsquigarrow y$ and $P \Vdash x \rightsquigarrow z$ then $y = z$.
- \simeq is a congruence.
- If P and Q are ill-formed then $P \simeq Q$.
- $\equiv \subseteq \simeq$.

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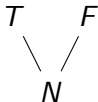
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Definitions

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- We pose $T = 10$, $F = 01$, $N = 00$ et $E = 11$.
NULL = $\{N\}$, VALID = $\{T, F\}$, FAULT = $\{E\}$.

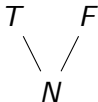
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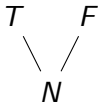
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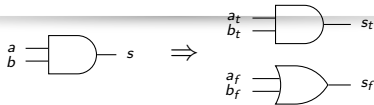
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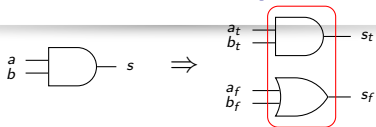
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- We extend these definitions to words of even length.
- For $u \in \Sigma^*$, we let $[u] \in \text{VALID}^*$ be the **corresponding word in dual-rail logic**.
example: $[0110] = FTTF = 01101001$

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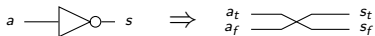
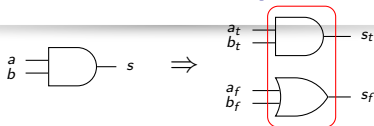
Transformation process



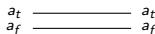
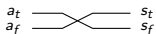
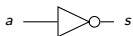
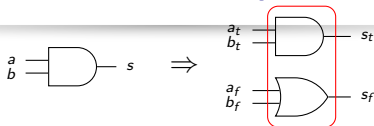
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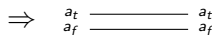
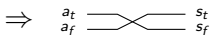
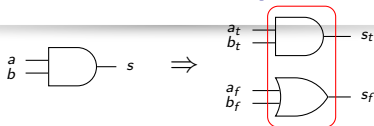
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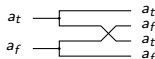
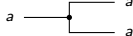
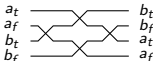
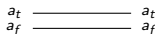
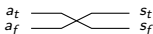
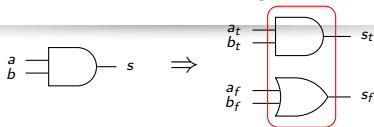
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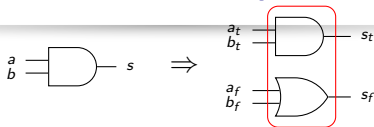
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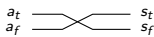
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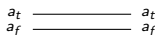
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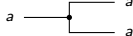
\Rightarrow



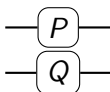
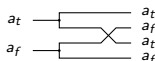
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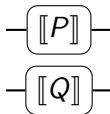
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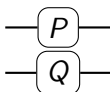
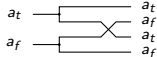
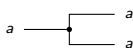
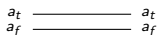
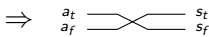
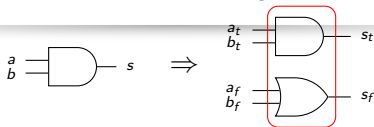
\Rightarrow



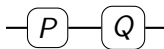
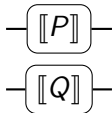
\Rightarrow



Transformation process



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Properties

$\llbracket P \rrbracket$ fulfils the DPL invariants.

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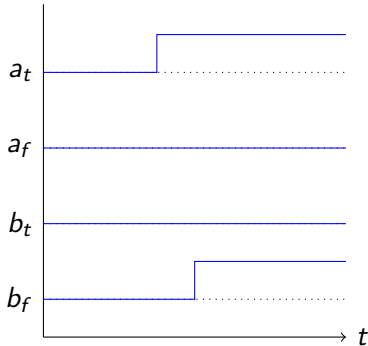
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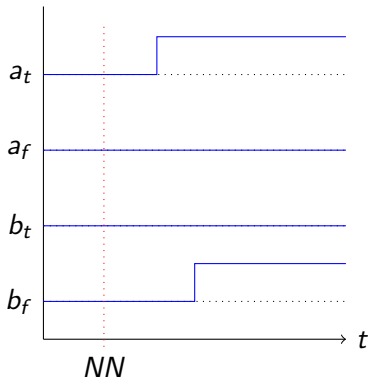
Interpretation of \preceq

\preceq models the transition of signals from precharge state (NULL) to evaluation state (VALID).



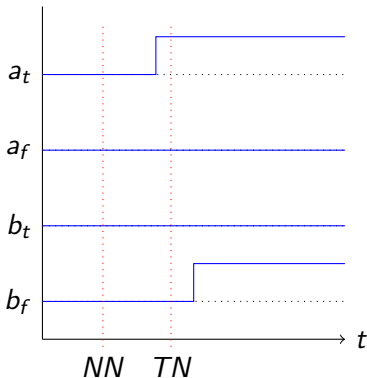
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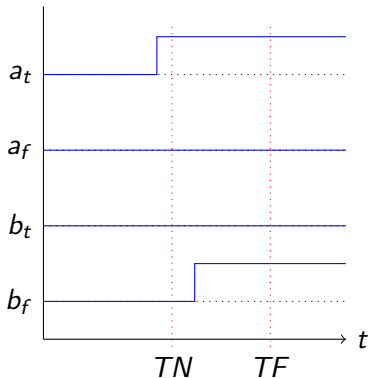
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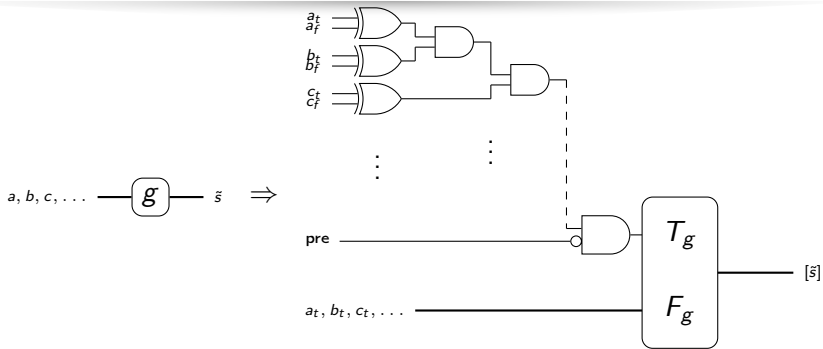
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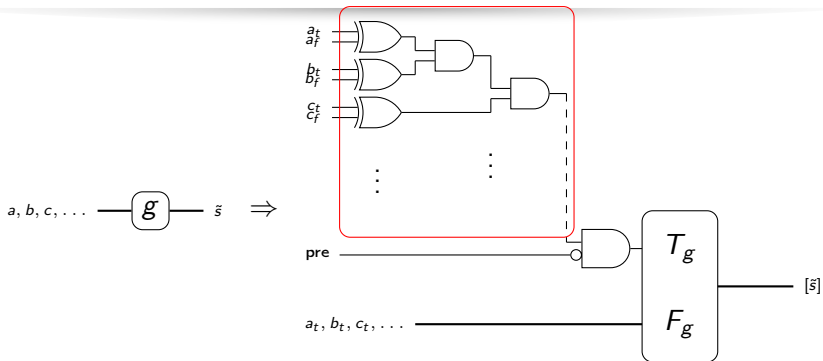
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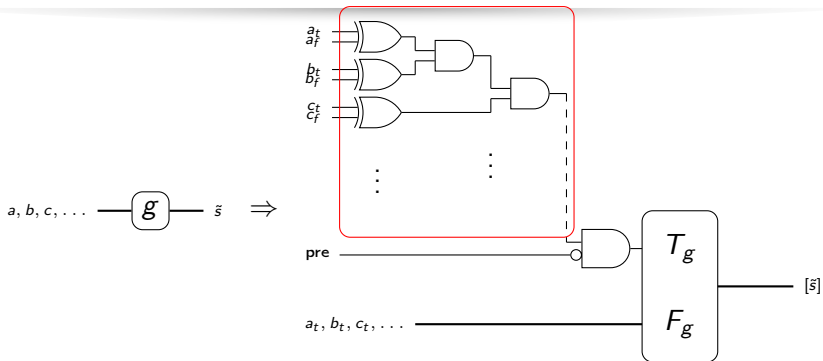
Transformation process



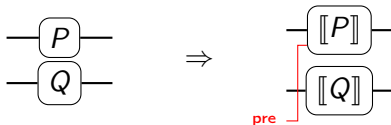
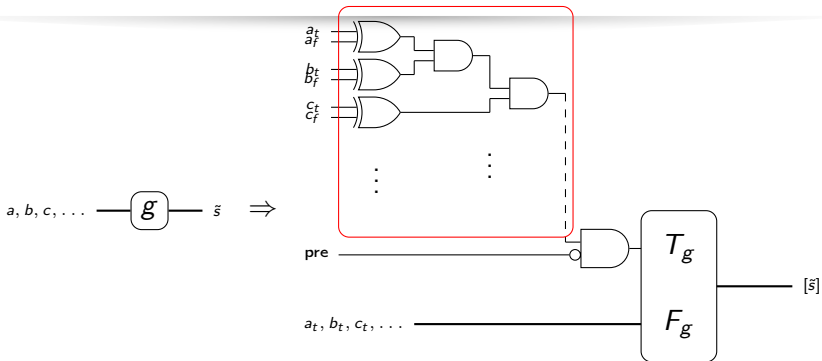
Transformation process



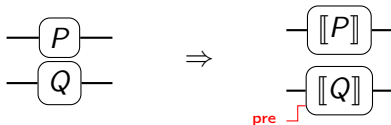
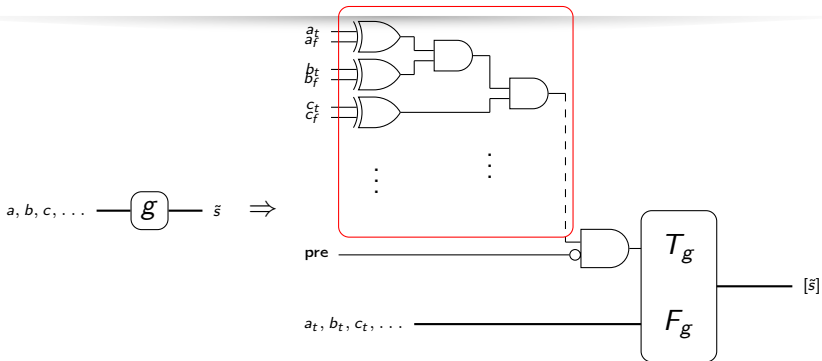
Transformation process



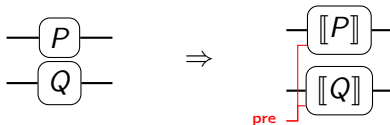
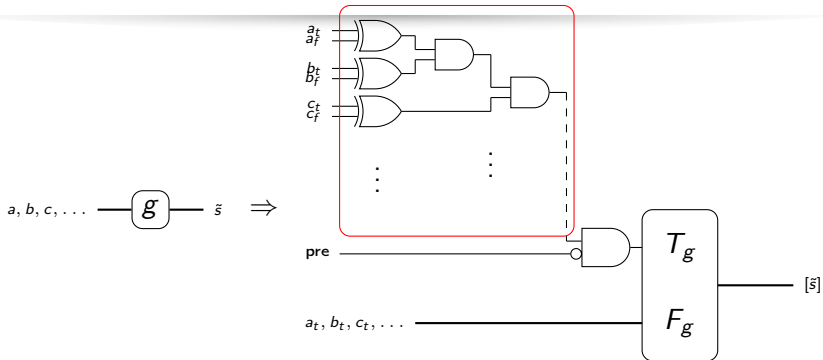
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If $\llbracket P \rrbracket \Vdash \hat{p}x \rightsquigarrow \hat{p}y$, $x \preceq x'$ and $\llbracket P \rrbracket \Vdash \hat{p}x' \rightsquigarrow \hat{p}y'$ then $y \preceq y'$.

Properties

There is no early-evaluation.

If $\llbracket P \rrbracket \Vdash \hat{0}x \rightsquigarrow y$ and $y \in \text{VALID}^*$ then $x \in \text{VALID}^*$.
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If $\llbracket P \rrbracket \Vdash \hat{O}x \rightsquigarrow y$ and $y \in \text{VALID}^*$ then $x \in \text{VALID}^*$.
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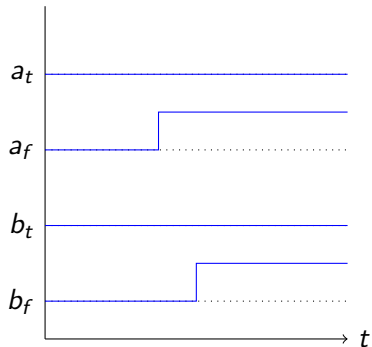
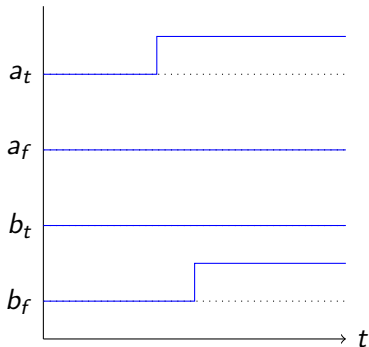
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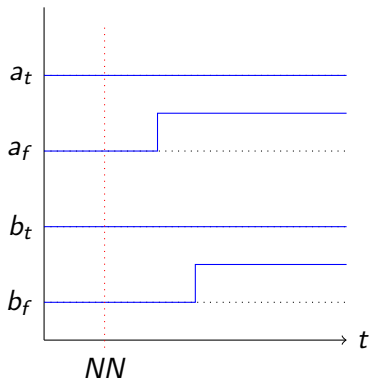
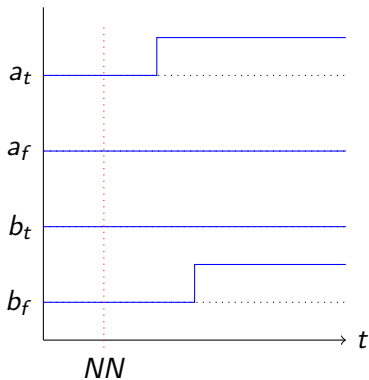
Interpretation of \sim

\sim equates words which have the same amount of information, i.e. in which corresponding dual-rail signals have the same nature.



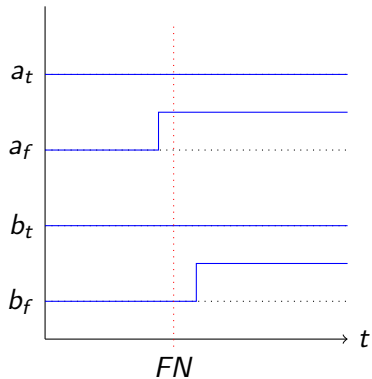
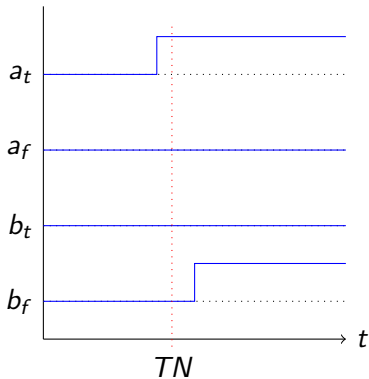
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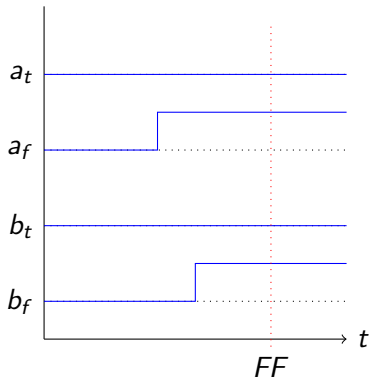
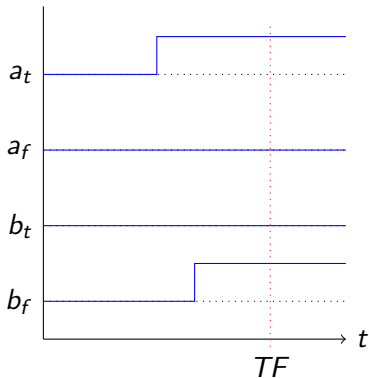
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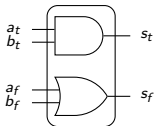
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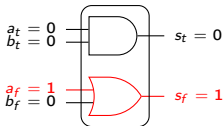
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Back to WDDL



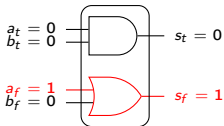
Back to WDDL



The AND_{WDDL} gate suffers from early-evaluation.

$x = 0100 \notin \text{VALID}^*$ and $y = 01 \in \text{VALID}^*$

Back to WDDL



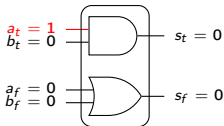
The AND_{WDDL} gate suffers from early-evaluation.

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The AND_{WDDL} gate behaves differently on equivalent inputs.

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Back to WDDL



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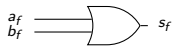
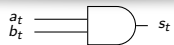
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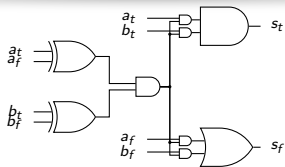
$x = 0100 \sim x' = 1000$ and $y = 01 \approx y' = 00$

1. Introduction
2. Combinational Circuits
 1. Language
 2. Formal semantics, equivalences
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 1. Preliminaries
 2. WDDL
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Discussion

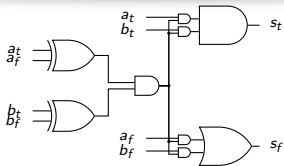


Discussion



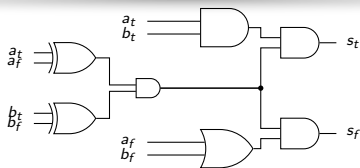
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Discussion



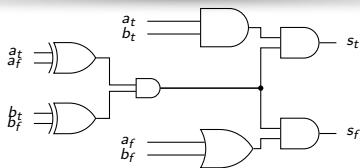
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- How to address the race between the synchronisation signal and the data signals? (DRSL vulnerability)
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Measure the activity of circuits and show that the activity of a circuit is constant on equivalent inputs, i.e.

$$x \sim x' \Rightarrow \mu_C(x) = \mu_C(x')$$

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 2. WDDL
 3. BCDL
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- Regarding security properties, we identified some necessary conditions to fulfil.

Perspectives

- Apply the model to other dual-rail styles.

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- Refine the model.

The End

Thank You

Rotations

On words

- $\vec{\epsilon} = \epsilon$ and, for $a \in \Sigma, u \in \Sigma^*, \vec{ua} = au$
- $\overleftarrow{\epsilon} = \epsilon$ and, for $a \in \Sigma, u \in \Sigma^*, \overleftarrow{au} = ua$

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We define by induction on $n \in \mathbb{N}$ the circuit ror_n :

$$\text{ror}_0 := \mathbf{0}$$

$$\text{ror}_1 := \mathbf{1}$$

$$\text{ror}_{n+2} := (\mathbf{1}^n \mid \mathbf{X}); (\text{ror}_{n+1} \mid \mathbf{1})$$

We have

$$\text{ror}_n \Vdash x \rightsquigarrow y \iff |x| = n \wedge y = \vec{x}$$

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We define by induction on $n \in \mathbb{N}$ the circuit rol_n :

$$\text{rol}_0 := \mathbf{0}$$

$$\text{rol}_1 := \mathbf{1}$$

$$\text{rol}_{n+2} := (\text{rol}_{n+1} \mid \mathbf{1}); (\mathbf{1}^n \mid \mathbf{X})$$

We have

$$\text{rol}_n \Vdash x \rightsquigarrow y \iff |x| = n \wedge y = \overleftarrow{x}$$

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We also have that:

- $\text{ror}_n; \text{rol}_n \equiv \mathbf{I}^n$
- $\text{rol}_n; \text{ror}_n \equiv \mathbf{I}^n$

Interleaving

On words

$\epsilon \parallel \epsilon := \epsilon$ and for $a, b \in \Sigma$, $u, v \in \Sigma^*$, $(au) \parallel (bv) := ab(u \parallel v)$

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We define by induction on $n \in \mathbb{N}$ the circuit int_n :

$$\text{int}_0 := \mathbf{0}$$

$$\text{int}_{n+1} := (\mathbf{1} \parallel \text{ror}_{n+1} \parallel \mathbf{1}^n); (\mathbf{1} \parallel \mathbf{1} \parallel \text{int}_n)$$

We have

$$\text{int}_n \Vdash x \rightsquigarrow y \iff x = u \bullet v \wedge |u| = |v| = n \wedge y = u \parallel v$$

Interleaving

On words

$\epsilon \parallel \epsilon := \epsilon$ and for $a, b \in \Sigma$, $u, v \in \Sigma^*$, $(au) \parallel (bv) := ab(u \parallel v)$

We define by induction on $n \in \mathbb{N}$ the circuit uint_n :

$$\text{uint}_0 := \mathbf{0}$$

$$\text{uint}_{n+1} := (\mathbf{1} \parallel \mathbf{1} \parallel \text{uint}_n); (\mathbf{1} \parallel \text{rol}_{n+1} \parallel \mathbf{1}^n)$$

We have

$$\text{uint}_n \Vdash x \rightsquigarrow y \iff y = u \bullet v \wedge |u| = |v| = n \wedge x = u \parallel v$$

Interleaving

On words

$\epsilon \parallel \epsilon := \epsilon$ and for $a, b \in \Sigma$, $u, v \in \Sigma^*$, $(au) \parallel (bv) := ab(u \parallel v)$

We also have that:

- $\text{int}_n ; \text{unint}_n \equiv \mathbb{I}^{2n}$
- $\text{unint}_n ; \text{int}_n \equiv \mathbb{I}^{2n}$